Predicting Glass Ribbon Shape in the Tin Bath

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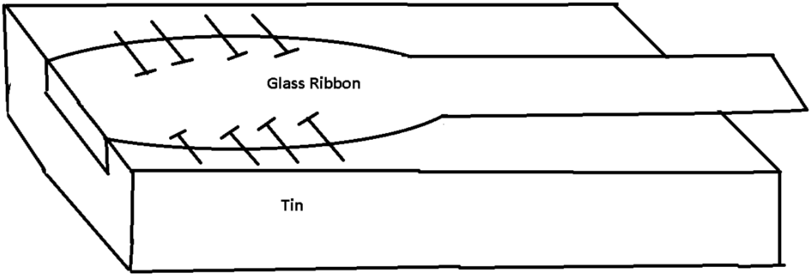
Outline

Glass Ribbon on tin bath



Physical Problem

Issues:

- Control
- Input flow and temperature
- Pulling speed
- Cooling and heating of tin
- Top roller,speed and angle
- Output,a uniformly flat glass with prescribed thickness

Mathematical Model

1. Temperature distribution
2. Tin bath flow
3. Glass flow

- Consider only 3

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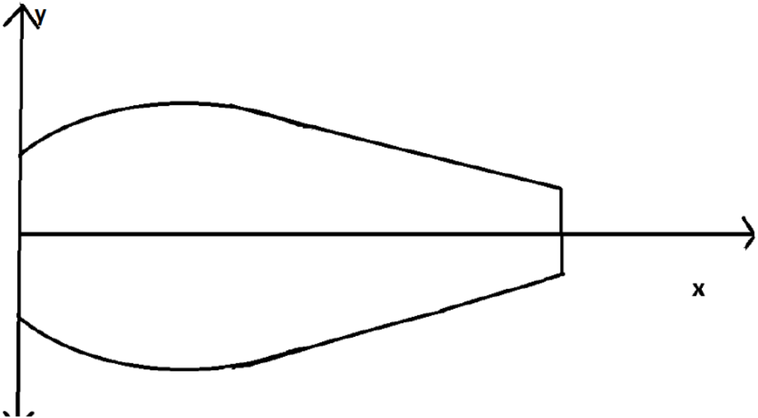
Glass Model

- Glass is a Newtonian fluid
- Nonuniform viscosity
- This model follows work in the PhD thesis by Peter Howell.

Glass Ribbon



Variables:

$$\mathbf{q} = u(x,y,z)\mathbf{i} + v(x,y,z)\mathbf{j} + w(x,y,z)\mathbf{k}, \quad \textit{velocity} \tag{1}$$

$$p(x,y,z), \qquad \textit{pressure} \tag{2}$$

$$h(x,y), \qquad \textit{height} \tag{3}$$

$$H(x,y), \quad \textit{geometric} \quad \textit{centre} \quad \textit{line} \tag{4}$$

$$\mu(x,y), \quad \textit{given} \quad \textit{viscosity} \tag{5}$$

Navier-Stokes:slow viscous flow

Navier-Stokes equation,slow viscous flow:

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \tag{6}$$

$$0 = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial v}{\partial z}\right) \tag{7}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left(\mu \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial w}{\partial z}\right) - \rho g \tag{8}$$

Boundary Condition

$\mathbf{z} = \mathbf{H} + \frac{1}{2}\mathbf{h}$

- No shear stress
- No normal stress
- kinematic condition

$\mathbf{z} = \mathbf{H} - \frac{1}{2}\mathbf{h}$

- no shear stress
- $normal \quad stress = hydrostatic \quad pressure \quad in \quad tin$
- kinematic condition

Lubrication Model

Dimensionless Variables:

$$x = L\bar{x}, \quad y = L\bar{y}, \quad z = \epsilon L\bar{z}, \tag{9}$$

$$u = U\bar{u}, \quad v = U\bar{v}, \quad w = \epsilon U\bar{w}, \tag{10}$$

$$p = \rho g \epsilon L\bar{p} \quad (Hydrostatic) \tag{11}$$

Lubrication model

- Nondimensionalised
- Perturbation expansion

$$\bar{u} = u_o + \epsilon^2 u_1 + \cdots \quad (12)$$

- Collect terms (powers of ϵ)
- Solved lowest order problem

Lowest order of ϵ :

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \tag{13}$$

$$\frac{\partial}{\partial \bar{z}} \left(\mu \frac{\partial \bar{u}}{\partial \bar{z}} \right) = 0 \tag{14}$$

$$\frac{\partial}{\partial \bar{z}} \left(\mu \frac{\partial \bar{v}}{\partial \bar{z}} \right) = 0 \tag{15}$$

$$\frac{-\partial \bar{p}}{\partial \bar{z}} + \frac{1}{A} \frac{\partial}{\partial \bar{z}} \left(\mu \frac{\partial \bar{w}}{\partial \bar{z}} \right) - 1 = 0 \tag{16}$$

Boundary conditions at $\mathbf{z} = \mathbf{H} + \frac{1}{2}\mathbf{h}$

$$\frac{\partial u}{\partial z} = 0 \tag{17}$$

$$\frac{\partial v}{\partial z} = 0 \tag{18}$$

$$w - \frac{\partial H}{\partial t} - u\frac{\partial H}{\partial x} - v\frac{\partial H}{\partial y} - \frac{1}{2}\frac{\partial h}{\partial t} - \frac{1}{2}u\frac{\partial h}{\partial x} - \frac{1}{2}v\frac{\partial h}{\partial y} = 0 \tag{19}$$

$$-p + \frac{\mu}{A}\frac{\partial w}{\partial z} - \frac{\mu}{A}\left(\frac{\partial H}{\partial x} + \frac{1}{2}\frac{\partial h}{\partial x}\right)\frac{\partial u}{\partial z} - \frac{\mu}{A}\left(\frac{\partial H}{\partial y} + \frac{1}{2}\frac{\partial h}{\partial y}\right)\frac{\partial v}{\partial z} = 0 \tag{20}$$

Boundary conditions at $\mathbf{z = H - \frac{1}{2}h}$

$$\frac{\partial u}{\partial z} = 0 \tag{21}$$

$$\frac{\partial v}{\partial z} = 0 \tag{22}$$

$$w - \frac{\partial H}{\partial t} - u\frac{\partial H}{\partial x} - v\frac{\partial H}{\partial y} - \frac{1}{2}\frac{\partial h}{\partial t} - \frac{1}{2}u\frac{\partial h}{\partial x} - \frac{1}{2}v\frac{\partial h}{\partial y} = 0 \tag{23}$$

$$-p + \frac{\mu}{A}\frac{\partial w}{\partial z} - \frac{\mu}{A}\left(\frac{\partial H}{\partial x} + \frac{1}{2}\frac{\partial h}{\partial x}\right)\frac{\partial u}{\partial z} - \frac{\mu}{A}\left(\frac{\partial H}{\partial y} + \frac{1}{2}\frac{\partial h}{\partial y}\right)\frac{\partial v}{\partial z} = (H - \frac{1}{2}h)\frac{\rho_{tin}}{\rho} \tag{24}$$

Lubrication Model

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0, \tag{25}$$

$$\frac{\partial}{\partial x}\left[2h\mu\left(2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[h\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] = \frac{h}{A}\left(1 - \frac{\rho_{tin}}{\rho}\right)\frac{\partial h}{\partial x}, \tag{26}$$

$$\frac{\partial}{\partial x}\left[2h\mu\left(2\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[h\mu\left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)\right] = \frac{h}{A}\left(1 - \frac{\rho_{tin}}{\rho}\right)\frac{\partial h}{\partial y}, \tag{27}$$

$$\tag{28}$$

$$H = \left(\frac{1}{2} - \frac{\rho_{tin}}{\rho}\right)h \tag{29}$$

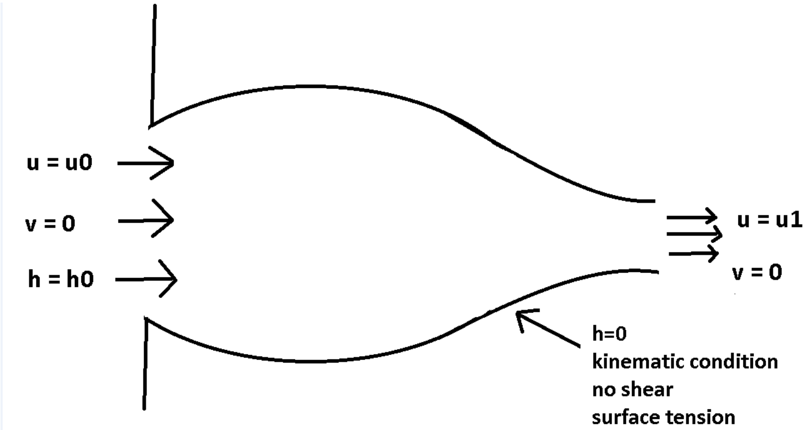
$$A = \frac{\rho g \epsilon L^2}{\mu_0 U} \tag{30}$$

Second order

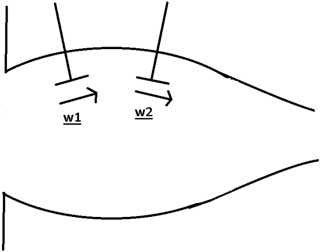
- Fredholm alternative

Determination of $u(x, y)$ and $v(x, y)$

Boundary conditions for ribbon



Extension for top rollers



$$\textit{partial} \quad \textit{differential} \quad \textit{equations} + \mathbf{F} \tag{31}$$

$$\mathbf{F} = \mu\beta(\mathbf{w} - \mathbf{q}) \tag{32}$$

Summary

- Determine partial differential equations for ribbon height
- Discussed free-boundary problem for h
- Extension for top rollers
- Implementation of number crunching still to be done

Thank You